Analogical Proportions and Creativity: A Preliminary Study

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Abstract

Analogical proportions are statements of the form "a is to b as c is to d", which express that the comparison of the elements in pair (a, b) and in pair (c, d) yield similar results. Analogical proportions are creative in the sense that given 3 distinct items, the representation of a 4th item d, distinct from the previous items, which forms an analogical proportion with them, can be calculated, provided certain conditions are met. After providing an introduction to analogical proportions and their properties, the paper reports the results of an experiment made with a database of animal descriptions and their class, where we try to "create " new animals from existing ones, retrieving rare animals such as platypus. We perform a series of experiments using word embeddings as well as Boolean features in order to propose novel animals based on analogical proportions, comparing the symbolic approach with non-contextual embeddings showing that such word embeddings obtain also good results. Furthermore we provide a series of experiments with sentence embeddings using contextual embeddings which are less conclusive. Finally, we also propose a more sophisticated creative process based on analogical proportions where the set of pairs (a, b) used is chosen and enlarged by means of a property-preserving operation.

Introduction

Creativity has raised interest for a long time in computer sciences and in AI (Boden 2004; Schmidhuber 2010; Colton 2008) with applications in many areas. Analogical reasoning has always been known to foster creativity, especially in creative thinking and problem solving (Holyoak and Thagard 1995; Goel 1997; Veale 2006). Indeed analogical reasoning makes a parallel between two situations, which suggests that what is true or applicable in the first situation might be true or applicable as well in the second situation which presents some similarity with the first one.

Analogical proportions (Prade and Richard 2021a) are quaternary relations denoted $a : b :: c : d$ between four items a, b, c, d, which read "a is to b as c is to d". In the following, a, b, c, d are represented by means of vectors ; these vectors may either be made of feature values, or be word embeddings (Mikolov et al. 2013). Analogical proportions can be viewed as a building block of analogical reasoning. Indeed they draw parallels between the ordered pairs (a, b) and (c, d). For example "*the calf is to the cow as the foal is to the mare*" put bovidae on a par with equidae. In such analogical proportions, the four items can be described by means of *the same set of features*. Analogical proportions have been successfully applied to classification (Miclet, Bayoudh, and Delhay 2008; Bounhas, Prade, and Richard 2017; Bounhas and Prade 2023; 2024), in preference prediction (Fahandar and Hüllermeier 2018; Bounhas et al. 2019), or for solving Raven IQ tests (Correa Beltran, Prade, and Richard 2016; Ragni and Neubert 2014).

(Mikolov et al. 2013) showed that embeddings language models have the potential to respect analogical proportions in a vector space, although later approaches showed that this was due to the limited corpus of analogies that was used proposing better resources for testing analogies (Gladkova, Drozd, and Matsuoka 2016; Wijesiriwardene et al. 2023). Recently, (Hu et al. 2023) have showed that Large Language Models have the capacity to solve Raven problems, provided to them in a natural language description, performing at least at the same level as human beings. To our knowledge, the creative capacities of analogies to produce something new have not been explored, especially in natural language.

This paper presents an investigation of the creative power of analogical proportions. This power relies on their capabilities to produce a fourth item from three items (provided that some conditions hold) as we shall see. The paper is organized into two main sections. The first one provides the necessary information on analogical proportions, as well as a new advanced procedure for guided creativity. The second one proposes a series of experiments using the Zoo dataset[1]. Initially, we generate symbolic animal descriptions from existing ones and check if these descriptions already exist in the database. We then conduct two other sets of experiments: one using vector representations of words, the other handling descriptive sentences. These experiments use word embeddings, either through static embeddings with the GloVe framework (Pennington, Socher, and Manning 2014) or contextual sentence embeddings with sBERT (Devlin et al. 2019; Reimers and Gurevych 2019). This is completed by a Raven Matrix style example, solved by means of analogical proportions, and a conclusive discussion.

[1]https://archive.ics.uci.edu/dataset/111/zoo

Analogical proportions

This section is structured in five subparts i) recalling the Boolean logical modeling, postulates, and properties of analogical proportions (AP), ii) providing an example showing their creative power, iii) handling nominal attribute values, iv) introducing nested APs, and v) dealing with word APs.

Truth table, postulates and properties

A logical modeling of an AP "a is to b as c is to d" where a, b, c, d are Boolean variables is given by the following formula that says that a *differs* from b as c *differs* from d and b *differs* from a as d *differs* from c" (Miclet and Prade 2009):

$$a : b :: c : d = ((a \wedge \neg b) \equiv (c \wedge \neg d)) \wedge ((\neg a \wedge b) \equiv (\neg c \wedge d))$$

This expression is true for the 6 patterns given in Table 1 and false for the $2^4 - 6 = 10$ other possible patterns.

a	b	c	d
0	0	0	0
1	1	1	1
0	0	1	1
1	1	0	0
0	1	0	1
1	0	1	0

Table 1: Boolean patterns making $a : b :: c : d$ true

This is the minimal Boolean model that satisfies the three following basic postulates (inspired from numerical proportions) that an AP should obey (Prade and Richard 2018):

- *reflexivity*: $a : b : a : b$;
- *symmetry*: $a : b :: c : d \Rightarrow c : d :: a : b$;
- *central permutation*: $a : b :: c : d \Rightarrow a : c :: b : d$.

As a consequence, we have the properties:

- $a : a : b : b$ (identity),
- $a : b :: c : d \Rightarrow b : a :: d : c$ (internal reversal), and
- $a : b :: c : d \Rightarrow d : b :: c : a$ (external permutation).

Remarkably enough, Boolean APs are *code independent*, i.e., $a : b :: c : d \Rightarrow \neg a : \neg b :: \neg c : \neg d$. Thus any property used for describing items can be encoded positively or negatively.

We assume that the items considered are represented by Boolean *vectors* with n components corresponding to n feature values, i.e., $\vec{a} = (a_1, ..., a_n)$, etc. An analogical proportion "$\vec{a}$ is to $\vec{b}$ as $\vec{c}$ is to $\vec{d}$", denoted $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$, is defined componentwise: $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$ iff $\forall i \in [1, n], a_i : b_i :: c_i : d_i$

Analogical proportions are creative

As an illustration, let us consider the geometric analogy problem in Figure 1 below (Correa Beltran, Prade, and Richard 2016; Prade and Richard 2021a). It can be encoded with five Boolean predicates: *hasRectangle (hR), hasBlackDot (hBD), hasTriangle (hT), hasCircle (hC), hasEllipse (hE)*, in that order in Table 2, where a, b, c are encoded. Each column is an AP equation $a_i : b_i :: c_i : x_i$.

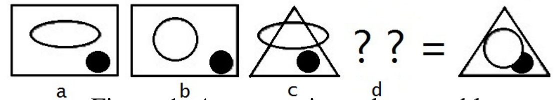

Figure 1: A geometric analogy problem

An equation $a : b :: c : x$ *has not always a solution*. Indeed the equations $1 : 0 :: 0 : x$, and $0 : 1 :: 1 : x$ do not have a solution so that the analogical proportion holds as one of the 6 patterns making an AP true. If $a : b :: c : x$ has a solution, it is *unique*. It is given by $x = c \equiv (a \equiv b)$, where $\equiv$ denotes the equivalence connective. This is the case in the example, where $\vec{x} = (01110)$ in Tab. 2 to $\vec{x} = \vec{d}$, drawn on the right of Fig. above.

It should be emphasized that here the description of $\vec{d}$ is computed directly from those of $\vec{a}, \vec{b}, \vec{c}$, which contrasts with this kind of (easy) IQ tests where the answer is to be found in a set of several candidate solutions. Indeed the first program for solving such problems (Evans 1968), in the early years of AI, was selecting the solution among candidate solutions on the basis of a distance accounting for the similarity of transformations for going from $\vec{a}$ to $\vec{b}$ and from $\vec{c}$ to $\vec{x}$ (where $\vec{x}$ is the considered candidate solution). The fact that the description of $\vec{x}$ is now directly computed from $\vec{a}, \vec{b}, \vec{c}$ shows the *creative power* of the analogical proportion. Moreover *$\vec{d} = \vec{x}$ is distinct from $\vec{a}$, $\vec{b}$ and $\vec{c}$.* This is general in $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$ as soon as $\vec{a}$ is distinct from $\vec{b}$ and distinct from $\vec{c}$ (on at least two attributes).

	hR	hBD	hT	hC	hE
$\vec{a}$	1	1	0	0	1
$\vec{b}$	1	1	0	1	0
$\vec{c}$	0	1	1	0	1
$\vec{x}$	?	?	?	?	?

Table 2: Encoding the example. $\vec{x} = (01110)$

In this example, for simplicity, the encoding only refers to the presence or absence of geometric shapes without taking care of their relative positions. However, in such a case, it would be possible to apply the same mechanism to a representation of the pictures at the pixel level, thus accounting for exact positions; see (Correa Beltran, Prade, and Richard 2016) for a discussion.

Nominal values

The description of items may involve nominal attributes, i.e., attributes with a finite domain with cardinality larger than 2. Then $a : b :: c : d$ holds for nominal variables iff:

$$(a, b, c, d) \in \{(s, s, s, s), (s, t, s, t), (s, s, t, t) | \; s, t \in \mathcal{A}\}$$

where s, t stand for any value of the attribute domain $\mathcal{A}$, (as first suggested in (Pirrelli and Yvon 1999), see also (Bounhas, Prade, and Richard 2017). It generalizes the Boolean case and preserves all the properties reported for this case.

An example of *nested* analogical proportion

We now illustrate nominal values with an example of AP (from (Klein 1982)) between four sentences $\boldsymbol{a}$ = "girls hate light", $\boldsymbol{b}$ = "boys love light", $\boldsymbol{c}$ = "women hate dark", $\boldsymbol{d}$ = "men love dark", viewed as ordered tuples of nominal values, with respect to the 3 attributes 'subject', 'verb', 'complement'. Each sentence is thus a vector with 3 components:

	subject	verb	complement
$\vec{a}$	girls	hate	light
$\vec{b}$	boys	love	light
$\vec{c}$	women	hate	dark
$\vec{d}$	men	love	dark

Table 3: Analogical proportion between sentences

Obviously, the values in the 'subject' column of Table 3 do not make an analogical proportion by themselves, since four distinct values are involved (while `hate : love :: hate : love`, and `light : light :: dark : dark` are clear analogical proportions in terms of nominal values). However `girls` : `boys` :: `women` : `men` can be viewed as a compact writing of an AP between descriptions in terms of a collection of Boolean features as in Table 4, where an AP holds in each column. So $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$, and thus $\boldsymbol{a} : \boldsymbol{b} :: \boldsymbol{c} : \boldsymbol{d}$ do hold. The AP $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$ is a *nested* one since it involves an attribute whose four values do not form one of the three basic patterns of a nominal analogical proportion, while those values are associated with vector descriptions which themselves form an AP.

	male	*female*	*young*	*adult*	*human*
`girls`	0	1	1	0	1
`boys`	1	0	1	0	1
`women`	0	1	0	1	1
`men`	1	0	0	1	1

Table 4: Girls are to boys as women are to men

Word analogical proportions

In the example of Table 3, the values of the vector components are words. Words can themselves be represented by vector embeddings (Mikolov et al. 2013). Analogical proportions can be directly defined in terms of such vector representations, as early foreseen in (Rumelhart and Abrahamson 1973). Then the analogical proportion $\vec{a} : \vec{b} :: \vec{c} : \vec{d}$ holds if and only if $\vec{a} - \vec{b} = \vec{c} - \vec{d}$, i.e., $\forall i, a_i - b_i = c_i - d_i$. Note that this agrees with the truth Table 1 in the Boolean case: when $a_i, b_i, c_i, d_i \in \{0, 1\}, a_i : b_i :: c_i : d_i \Leftrightarrow a_i - b_i = c_i - d_i$. See (Prade and Richard 2021b) for more details on this view. Thus, the vector embeddings of words 'girls', 'boys', 'women', and 'men' should be such that $v_{\vec{girls}} - v_{\vec{boys}} = v_{\vec{women}} - v_{\vec{men}}$.

Moreover this view agrees with the computation of vector embeddings of sentences as the sum of the vector embeddings of the words in the sentence. Indeed we have $(\vec{a}-\vec{b})+(\vec{a'}-\vec{b'}) = (\vec{c}-\vec{d})+(\vec{c'}-\vec{d'}) \Leftrightarrow (\vec{a}+\vec{a'})-(\vec{b}+\vec{b'}) = (\vec{c}+\vec{c'})-(\vec{d}+\vec{d'})$, which means that if two 4-tuples $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ and $(\vec{a'}, \vec{b'}, \vec{c'}, \vec{d'})$ of words are an AP, their additive grouping is also an AP. Indeed, in order to confirm that, we used GloVe vector embeddings representing each sentence as the mean of the embedding vectors of its words. We then calculated $1 - \cos \frac{\vec{b}-\vec{a}}{\vec{d}-\vec{c}}$ which represents how close the vectors are in forming an analogy, obtaining a value of 0.8513 showing that the 4 vectors are close to forming a parallelogram.

Guided creativity

Given a set $\mathcal{I}$ of existing items, each represented in terms of the same set of Boolean attributes $\mathcal{A}$, creativity may amount to produce a new item, not in $\mathcal{I}$, but described by the same set of attributes $\mathcal{A}$. Viewed like that, creativity looks easy: we may choose at random values for the attributes in $\mathcal{A}$ and check if the result is not already in $\mathcal{I}$. However, with such a process, we have no control on the attribute values that might be desirable.

In the previous section, we have proposed another option for generating a new item from three known items using analogical proportions. As emphasized in (Prade and Richard 2023), analogical proportion is a matter of pairing a pair $(\vec{a}, \vec{b})$ with a pair $(\vec{c}, \vec{d})$. Thus from the set of items $\mathcal{I}$, one can build a set of k ordered pairs $\mathcal{P} = \{(\vec{a}^j, \vec{b}^j) \mid \vec{a}^j \in \mathcal{I}, \vec{b}^j \in \mathcal{I}, j = 1, \cdots, k\}$. This set of pairs potentially represents the knowledge that can be obtained by pairwise comparison of the items in $\mathcal{I}$. Taking an item $\vec{c}$ as a starting point that we would like to modify, we can obtain a new item $\vec{x}$ by solving, component by component, the equations $\vec{a}^j : \vec{b}^j :: \vec{c} : \vec{x}^j$ (when the solution exists), i.e., namely $\vec{x^j} = \vec{c} \equiv (\vec{a}^j \equiv \vec{b}^j)$ for any pair j. In other words, we look for the set of solutions

$$\mathcal{S} = \{\vec{x} \mid \exists (\vec{a}^j, \vec{b}^j) \in \mathcal{P}, j \in \{1,.., k\} \text{ s.t. } \vec{a}^j : \vec{b}^j :: \vec{c} : \vec{x} \text{ holds}\}$$

Note that i) if a property is gained (resp. lost) when going from $\vec{a}^j$ to $\vec{b}^j$, i.e., $a_i^j = 0, b_i^j = 1$ (resp. $a_i^j = 1, b_i^j = 0$) and $\vec{c}$ has not the property (resp. has the property), then this property is also gained (resp. lost) by $\vec{x}^j$; ii) if there is no change from $\vec{a}^j$ to $\vec{b}^j$ on attribute i, i.e., $a_i^j = b_i^j$, then $\vec{x}^j$ will copy the value of $\vec{c}^j$ for the value of attribute i.

In order to better control the generation process of new items, one may use a set of *selected* pairs. More precisely, let us suppose that the items in $\mathcal{I}$ from which the set $\mathcal{P}$ of pairs is built, represent objects / profiles / situations belonging to a real world universe, and then, that each ordered pair $(\vec{a}^j, \vec{b}^j)$ of vectors, represents legitimate / feasible / allowed / valuable changes from $\vec{a}^j$ to $\vec{b}^j$.

When there is no solution in $\mathcal{S}$ (because $\mathcal{P}$ is now smaller), or when the items found are not considered satisfactory enough, we have to consider the option of enlarging the potential set $\mathcal{P}$ we start with, namely by building new ordered pairs from the pairs already in $\mathcal{P}$. This gives birth to a *creative inference process* that attempts to improve a particular item or entity, taking advantage of a set of ordered pairs of existing items, using the analogical proportion-based mechanism.

In order to enlarge the initial base of pairs, we compute new pairs by means of an operation, denoted $\wedge\!\!\vee$, introduced in (Prade and Richard 2023) that produces a new ordered pair by combining two pairs. This operation $\wedge\!\vee$ is defined in the following way:

$$(\vec{a}, \vec{b}) \wedge\!\vee (\vec{c}, \vec{d}) = (\vec{a} \wedge \vec{c}, \vec{b} \vee \vec{d})$$

where the conjunction and the disjunction of items are defined componentwise:

$\vec{a} \wedge \vec{b} = (a_1 \wedge b_1, ..., a_n \wedge b_n)$;
$\vec{a} \vee \vec{b} = (a_1 \vee b_1, ..., a_n \vee b_n)$.

Obviously, this operator $\wedge\!\vee$ is commutative, associative and idempotent by construction. It can be checked that if (a_i, b_i) or $(c_i, d_i) = (0, 1)$, $(a_i \wedge c_i, b_i \vee d_i) = (0, 1)$. Thus, if a property i is acquired when going from the first element of one of the pair to the second element of this pair, then the property is also acquired in the new pair resulting of the combination by $\wedge\!\vee$.[2] By contrast, $(a_i \wedge c_i, b_i \vee d_i) = (1, 0)$ only if $(a_i, b_i) = (c_i, d_i) = (1, 0)$. Thus, if a property i is lost in the new pair, this was already the case in each of the pairs combined. The operation $\wedge\!\vee$ has the merit of "cumulating" the acquisition of features by preserving patterns $(0, 1)$.

Extending the initial set $\mathcal{P}$ of pairs gives us more chance to create a new item of interest, perhaps with more desirable features. More precisely we compute the set $\mathcal{S}$ previously defined where $\mathcal{P}$ is replaced by $\mathcal{P}' = \{(a^k, b^k) | (a^k, b^k) = (a^i, b^i) \wedge\!\vee (a^j, b^j)$ s.t. $((a^i, b^i), (a^j, b^j)) \in \mathcal{P}^2\}$. We may apply this enlargement of $\mathcal{P}$ iteratively to $\mathcal{P}'$ and so on. Due to idempotency of $\wedge\!\vee$, $\mathcal{P} \subset \mathcal{P}'$.

Then given a current fixed item represented by a vector $\vec{c} \in \mathcal{I}$ one may look for what new item(s) that could be obtained by applying some change existing in the base of ordered pairs $\mathcal{P}'$.

We now illustrate the whole process with a small example freely inspired from creativity in literature. The items we consider are supposed to be sketchy descriptions of novels. We have 6 attributes that say if the novel i) is written in verse or in prose (`ver.`); ii) is epistolary or not (`epi.`) ; iii) is an adventure novel or not (`adv.`); iv) is a romance novel or not (`rom.`); v) takes place today or not (`tod.`); vi) takes place in an exotic country or not (`exo.`).

We have two pairs constituting $\mathcal{P}$:
- $(\vec{a_1}, \vec{b_1}) = ([0, 0, 0, 0, 1, 0], [0, 1, 0, 0, 1, 1])$
- $(\vec{a_2}, \vec{b_2}) = ([0, 0, 0, 0, 1, 0]), [1, 0, 0, 0, 1, 1])$

Thus $\vec{a_1}$ refers to a novel written in prose, non epistolary, with no adventure, no romance, with takes place today in a non exotic country. $\vec{b_1}$ is like $\vec{a_1}$ except it is epistolary and exotic and since $(\vec{a_1}, \vec{b_1}) \in \mathcal{P}$, this means that for the user $\vec{b_1}$ is regarded as "more interesting" than $\vec{a_1}$. The pair $(\vec{a_2}, \vec{b_2})$ can be read in a similar manner.

When we extend $\mathcal{P}$ with operator $\wedge\!\vee$ (but without doing the full closure), we add to $\mathcal{P}$ the following pair:
- $(\vec{a_3}, \vec{b_3}) = ([0, 0, 0, 0, 1, 0], [1, 1, 0, 0, 1, 1])$

[2]However note that $(0, 0) \wedge\!\vee (1, 1) = (1, 1) \wedge\!\vee (0, 0) = (0, 1)$, which may create some unsupported / unfeasible change; in such a case, it may be better to not consider the generated pair(s) in a further process.

since $(\vec{a_3}, \vec{b_3}) = (\vec{a_1}, \vec{b_1}) \wedge\!\vee (\vec{a_2}, \vec{b_2})$.
Because, $\vec{a_1} = \vec{a_2}$, obviously $\vec{a_1} = \vec{a_3}$, but this is not at all compulsory, just a matter of simplicity in the example. Note that $\vec{b_3}$ cumulates the "advantages" of $\vec{b_1}$ and $\vec{b_2}$ with respect to $\vec{a_1}$. Starting from $\vec{c} = [0, 0, 0, 1, 0, 0]$, we observe in Table 5 that the 3 corresponding analogical equations are solvable. Thus, we have obtained 3 new tentative profiles of novels $\vec{x_1}, \vec{x_2}, \vec{x_3}$. For instance, the solution of the third equation is then a new type of novel, which is distinct from the 5 existing vectors $\vec{a_1}, \vec{b_1}, \vec{b_2}, \vec{x_1}, \vec{x_2}$. It is an exotic romance novel both written in verse and epistolary, with no adventure, taking place in the past or in the future. See Table 5.

	`ver.`	`epi.`	`adv.`	`rom.`	`tod.`	`exo.`
$\vec{a_1}$	0	0	0	0	1	0
$\vec{b_1}$	0	1	0	0	1	1
$\vec{c}$	0	0	0	1	0	0
x_1	0	1	0	1	0	1
	`ver.`	`epi.`	`adv.`	`rom.`	`tod.`	`exo.`
$\vec{a_2}$	0	0	0	0	1	0
$\vec{b_2}$	1	0	0	0	1	1
$\vec{c}$	0	0	0	1	0	0
x_2	1	0	0	1	0	1
	`ver.`	`epi.`	`adv.`	`rom.`	`tod.`	`exo.`
$\vec{a_3}$	0	0	0	0	1	0
$\vec{b_3}$	1	1	0	0	1	1
$\vec{c}$	0	0	0	1	0	0
x_3	1	1	0	1	0	1

Table 5: $(\vec{a_3}, \vec{b_3}) = (\vec{a_1}, \vec{b_1}) \wedge\!\vee (\vec{a_2}, \vec{b_2})$.

The process we have described ensures that i) the obtained items are new, and ii) they are obtained from an existing $\vec{c}$ on the basis of already existing changes, since observed on ordered pairs of existing items. Are the results thus obtained valuable? This is a completely different issue: all that is new is not necessarily valuable, but at least it has the merit of being new, which can help you think outside the box.

In Table 5 we have only provided a very small toy example for explaining the mechanism that enables us i) to enlarge a set of pairs that presents interesting changes that are cumulated in the enlargement; ii) and then to produce a new improved item from the obtained set of ordered pairs and a reference item ; see (Prade and Richard 2024) for a preliminary implementation. Note that one way of studying the creative power of this machinery might be to compare the probability distribution of feature values in the pairs we start with, to the probability distribution of feature values in the set of elements produced (varying, or not, the reference element) using the Kullback-Leibler divergence (Kullback and Leibler 1951).

The analogical proportion-based inference provides an approach to creativity that is based on recopying of what is observed in a pair, in terms of change and permanence on another pair, whose first element is known. Moreover we have shown that it is possible to combine existing ordered pairs into new pairs while preserving desirable features. Still, we certainly do not claim that every form of creative analogical inference, taken in the broadest sense, could be captured by the transfer mechanism we propose.

Experiments

Procedure

We start with a database containing the descriptions of animals in terms of Boolean features. We also assume that a class is known for each animal. Thus, an animal A is described by a vector $\vec{a}$ together with its class $cl(\vec{a})$. The idea, given a subset S of the database, is to compute the solutions of equations of the form $\vec{a} : \vec{b} :: \vec{c} : \vec{x}$ where $\vec{a}, \vec{b}, \vec{c} \in S$, and to see if $\vec{x}$ is, or not, in the database. Also we may use only the restriction of vectors $\vec{a}, \vec{b}, \vec{c}$ to a subset of features. More precisely, we look for small subsets of features used for the description of animals, as in the example of Table 6 (where the solution indeed exists in the animal reign).

	$suckle\ their\ young$	$lay\ eggs$
$scorpions$	0	0
$mammals$	1	0
$birds$	0	1
?	1	1

Table 6: ? = *monotremes*

In this example, the analogical equation solving "creates" an animal species that both lays eggs and suckles her young. It turns out that such animals exist: platypus, echidnas.

Note that when looking for analogical solutions, we should take care of two issues: 1; the equation should be solvable for each feature considered; 2. $\vec{a} : \vec{b} :: \vec{c} : \vec{x}$ and $\vec{a} : \vec{c} :: \vec{b} : \vec{x}$ have the same solution if any. Moreover, from a creativity point of view the vectors $\vec{a}, \vec{b}, \vec{c}$ (or the sub-vectors used) refer to animals that should be sufficiently different. This is why it may be useful to enforce the constraints $cl(\vec{a}) \neq cl(\vec{b}), cl(\vec{a}) \neq cl(\vec{c}), cl(\vec{b}) \neq cl(\vec{c})$.

Besides, we are also interested in solving analogical equations when vectors are word embeddings, i.e., computing $\vec{x} = \vec{c} + \vec{b} - \vec{a}$, and looking for the words whose embedding is close to $\vec{x}$. The question is then to see if the results are compatible with those obtained from the Boolean representations.

16 features	5 most important features
22.64%	59.67%

Table 7: Precision results using Boolean vectors.

Implementation

Symbolic approach We use the *Zoo* dataset (see footnote 1). This dataset contains 101 animals each one described by 17 features and their class. Out of these 17 features, 15 are binary while two are nominal. The first nominal feature describes the number of legs that the animal has while the other one describes its class (mammal, reptile, etc).

	P@1	P@2	P@3	P@4	P@5	P@6	P@7	P@8
GloVe	46.33	64.60	73.94	78.75	81.33	82.60	83.21	83.44
exp.	6.87	12.48	16.65	20.66	24.04	27.17	30.15	32.89
not exp.	0.0	0.0	0.0	0.0	0.0	0.2	1.4	2.6
om.	0.0	0.01	0.08	0.12	0.19	0.2	0.4	0.6

Table 8: Precision@k for GloVe and sBERT vectors in percents. For sBERT results reflect the percentage of animals that are predicted that have also been predicted by the symbolic approach. Descriptions of animals explicitly mention the animal (exp.), or not (not exp.) or they do not explicitly mention the animal and omit features that the animals lack (om.)

Initial experiments took into account the full gamut of 16 features, excluding the class of the animal. For the full list of 101 animals, we calculated all possible triplets $\binom{101}{3}$ and kept only the ones for which the three animals belonged to a different class. For each triplet $(\vec{a}, \vec{b}, \vec{c})$ we predicted the existence, or not, of a fourth element representing a "potential" animal using analogical proportions for each feature, as described in the previous sections.

Imposing analogical proportions for the full 16 features is quite restrictive, since it suffices for one feature to not be in AP in order to discard the whole triplet. Examining all possible subsets of features of cardinality 2 or more is computationally prohibitive since this would result in $\binom{101}{3} \times 2^{15}$ instances. We wanted thus to identify the subset of the most *"important"* features in order to perform our experiments. In order to do so, we identified the features which had values shared by most animals and we selected the top five. These features were `hair, eggs, milk, venomous` and `domestic`. We applied the same procedure as before for this subset of features.

GloVe Embeddings In further experiments we investigated how embeddings representing words can cope with creativity using analogies. As explained above, each element in a triplet $(\vec{a}, \vec{b}, \vec{c})$ is represented in a vectorial form with real values. We predict a 4th element $\vec{d}$ such that $\vec{d} = \vec{c} + \vec{b} - \vec{a}$. We represent each animal A, B, C using both static and contextual embeddings. For the static embeddings we chose GloVe (Pennington, Socher, and Manning 2014) embeddings with 300 dimensions ("common crawl" with 840 billion tokens). After calculating $\vec{d}$ we find the words represented by the closest Glove embeddings to this vector using Euclidean distance. Our goal in this set of experiments was to examine whether these words represent animals that are indeed found in the Zoo database.

sBERT Embeddings Finally, we also wanted to test contextual embeddings created using Transformer architectures (Vaswani et al. 2017). We opted to use an approach based on the BERT architecture (Devlin et al. 2019). Transformer based architectures provide embeddings that are not static but instead depend on their context, relying heavily on an intricate attention mechanism on a fixed size context of sub-word tokens. For this reason, we couldn't follow the same procedure as the static embeddings of GloVe. Instead we opted to create sentences for each animal in the database, reflecting the features that describe each animal. More precisely, we opted for the aforementioned five features (`hair, eggs, milk, venomous` and `domestic`). For example, for *platypus* we created the following sentence, reflecting the features that describe this animal:

> *Platypus is an animal that has hair, lays eggs, produces milk, isn't venomous, isn't domestic.*

Each such created phrase was encoded using sBERT (Reimers and Gurevych 2019). Due to the non-static nature of BERT embeddings, we couldn't search among the potential sentence in the sBERT embedding space that was closest to the quantity $\vec{d} = \vec{c} + \vec{b} - \vec{a}$. We opted to do the following. Once we had created such embeddings, for a given triplet (A, B, C) of animals we calculated the Euclidean distance, for any $D \notin \{A, B, C\}$ in the database, between $\vec{a} - \vec{b}$ and $\vec{c} - \vec{d}$. We describe results in the following section.

The aforementioned descriptions of animals that we created include the animal itself. As such, sBERT sentence embeddings undoubtedly rely heavily on this information. We thus wanted to test whether sBERT sentence embeddings which do not use information on the animal are able to simulate the symbolic approach that we described above. We thus proceeded by creating descriptions of animals based on their features without mentioning the animal explicitly. We created two variations: a) full descriptions and b) descriptions omitting the absence of features. For example, for platypus the full description would be:

> *This is an animal that has hair, lays eggs, produces milk, isn't venomous, isn't domestic.*

while the description omitting absent features would be:

> *This is an animal that has hair, lays eggs, produces milk.*

As in the previous case, for a given triplet (A, B, C) we created descriptions and thus obtained sBERT vectors $\vec{a}, \vec{b}$ and $\vec{c}$ from which we calculated $\vec{d} = \vec{c} + \vec{b} - \vec{a}$. We then proceed in calculating for the 5 features mentioned above[3] $2^5 = 32$ different descriptions exactly in the same way as we did for the animals and obtained d_i sBERT vectors with $i \in [1 \ldots 32]$ for each one of the possible combination. We were thus able to calculate in a decreasing order the Euclidean distance between d_i and $\vec{d} = \vec{c} + \vec{b} - \vec{a}$. Furthermore, for each of the 32 d_is we could retrieve the subset of animals that shared the exact same characteristics. This allowed us to directly compare a BERT-based approach in relation to the symbolic approach that we described above. We describe our results in the following section.

Results and discussion

In the following we describe and comment on the results we obtained with the methods described above.

[3]That is for the following set of features: `hair, eggs, milk, venomous` and `domestic)`.

Symbolic approach

Evaluating creativity is still an open research subject. There are no widely accepted measures of creativity and most evaluations remain subjective. In this paper we wanted to have a rough estimate of how often the animals that we proposed did indeed exist in the micro-world of the Zoo database. We thus chose to measure the Precision of our results: number of predictions for which at least one animal exists in the Zoo database over all our predictions. The results for the Boolean vectors are shown in Table 7.

As we can see when we use the full list of 16 features, 22.64% of the proposed animals already exist in the Zoo database while for the 5 most important features 59.67% exist. Having in mind the subjectivity in evaluating creativity, let us examine some specific cases. Take for example the following three animals: *seasnake, frog, aardvark*. Their vectors for the subset of important features are the following $\vec{a} = (0, 0, 0, 1, 0), \vec{b} = (0, 1, 0, 1, 0), \vec{c} = (1, 0, 1, 0, 0)$ respectively. When we apply the APs on these vectors we obtain vector $\vec{d} = (1, 1, 1, 0, 0)$, predicting thus a hairy animal that lays eggs and milks their children. Although it would seem strange at first, such an animal indeed exists: the *platypus*. Let us consider now the triplet of animals (*platypus, antelope, stingray*) with corresponding vectors: $\vec{a} = (1, 1, 1, 0, 0), \vec{b} = (1, 0, 1, 0, 0), \vec{c} = (0, 1, 0, 1, 0)$. Applying the same procedure we obtain $\vec{d} = (0, 0, 0, 1, 0)$ which corresponds to a non-hairy venomous being that does not lay eggs (when giving birth) and does not milk their children. Such animals do exist: *scorpions* and *seasnakes*. Our approach does indeed identify both of them.

GloVe embeddings

Regarding GloVe, the computational cost was prohibitive for calculating $\binom{101}{3}$ instances. Since our goal in this paper is to explore the potential of analogies for creativity, we wanted to compare results between a predicate logic approach and one based on word embeddings. We thus decided to evaluate GloVe on the results that were obtained with the predicate logic based approach and see whether GloVe could match them. After calculating $\vec{d} = \vec{c} + \vec{b} - \vec{a}$ we look for the 10 closest GloVe vectors to $\vec{d}$ according to the Euclidean distance. We discard instances that do not correspond to animals according to WordNet Synsets (McCrae et al. 2019), as implemented by NLTK. We then calculate the precision at k, that is for the first k propositions of GloVe we examine whether at least one is present as an animal in the Zoo database, after stemming with NLTK. The results for $k \in [1, 8]$ are shown in Table 8. As we can see, already for $k = 2$ GloVe achieves a precision of 64.60 exceeding the results obtained in the predicate-logic based method, while for $P@8 = 83.44$.

BERT embeddings

Regarding contextual embeddings using Transformer architectures, as mentioned above we used sBERT (Reimers and Gurevych 2019) after creating sentences for each animal reflecting their features. Although we could calculate $\vec{d} = \vec{c} + \vec{b} - \vec{a}$ and then search for a sentence that is closer to $\vec{d}$, doing so for any potential phrase is computationally prohibitive. We opted to identify the $\vec{d}$ for vectors representing animals restricted to the Zoo database, by calculating Euclidean distances between $\vec{a} - \vec{b}$ and $\vec{c} - \vec{d}$ for any $d \notin \{a, b, c\}$. We then wanted to evaluate results from this approach in relation to the results from the symbolic approach described above. More precisely, for any triplet that the symbolic approach provided a non-empty list of animals, we calculated the $Precision@k$ between this list and the top $k = 8$ animals (represented by vector $\vec{d}$) calculated with sBERT. Results are shown in Table 8.

As we can see, there is some overlap between the BERT-based approach and the symbolic one but we cannot claim that there is a very close overlap. Having said that, let us note though that, as in the symbolic case, in the case of BERT, animals such as *platypus* are also "predicted". For example, for the triplet *(scorpion, pitviper, aardvark)* the symbolic approach provides *platypus* as the only prediction while the BERT-based approach provides once again *platypus* as the most probable "analogical" animal. The same happens for the triplet *(scorpion, aardvark, seawasp)*. Concerning *scorpion*, this is given as the most probable "analogical" animal by various triplets by sBERT, for example *(oryx, ostrich, pony)*, as well as the symbolic approach, for example *(platypus, opossum, seawasp)*, although there is no triplet that provides *scorpion* both for the symbolic and BERT-based approach at the same time.

A second approach consisted in comparing directly the symbolic approach in relation to a BERT-based one. We thus created, as described in the previous section, full descriptions or descriptions omitting absent characteristics of animals without providing the name of the animals. This allowed us to generate sBERT vectors and calculate $\vec{d} = \vec{c} + \vec{b} - \vec{a}$. We then proceeded in creating similar descriptions for all possible combination of the 5 selected features yielding thus $2^5 = 32$ descriptions and thus 32 sBERT vectors $\vec{d_i}, i \in [1, \ldots, 32]$. We then ordered in a decreasing order each $\vec{d_i}$ in relation to $\vec{d}$. Since we could retrieve the subset of animals that satisfy each description we could thus directly compare to the symbolic approach. We decided to use the $Precision@k$ as previously in order to measure the overlap between symbolic and BERT-based approach. Since we had full descriptions as well as descriptions that omit features that the animal lacks, we present our results for each series of experiments in Table 8. As we can see, when no animals are explicitly mentioned the results between BERT and the symbolic approach are almost disjoint. As the results from Table 8 show, BERT relies heavily on the semantics of the word itself describing the animal instead of the description of its characteristics.

A last example of creativity with AP's

Raven tests (Raven 1965) are well-known IQ tests. They take the form of a series of instances having the format of a $n \times n$ matrix (where n is 2, 3 or 4) whose cells contain diverse geometric figures (See Figure 2 for an example), except the last cell which is empty and has to be completed by selecting a solution among 8 candidates(when $n = 3$). The series of matrices in a test are progressive in difficulty (then the denomination of Raven's Progressive Matrices, abbreviated as RPM). When no candidate solution is provided, we face a creative exercise for building the contents of the empty cell!

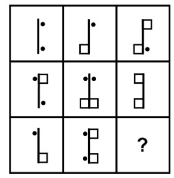

Figure 2: The problem

Let us denote our set of images, where $im9$ has to be guessed, as a matrix such as:

$$\begin{pmatrix} im1 & im2 & im3 \\ im4 & im5 & im6 \\ im7 & im8 & im9 \end{pmatrix}$$

Each image i can be encoded by a vector with 4 components:

- tl describes the item on Top Left of the vertical bar;
- tr describes the item on Top Right of the bar;
- br describes the item on Bottom Right of the bar;
- bl describes the item on Bottom Left of the bar.

The value of each component belongs to the set $X = \{dot, square, nothing\} = \{d, s, n\}$ so that an image is a vector in X^4.

This leads to a new representation of the initial matrix as:

$$\begin{pmatrix} nddn & ndns & nsds \\ dsdn & ddss & snns \\ dnsn & dssd & im9 \end{pmatrix}$$

In terms of analogical proportion, the problem can be read as the AP's: $(im1, im2) : im3 :: (im4, im5) : im6$ and $(im4, im5) : im6 :: (im7, im8) : im9$, or with the vector representation: $(nddn, ndns) : nsds :: (dsdn, ddss) : snns$ and $(dsdn, ddss) : snns :: (dnsn, dssd) : im9$, $im9$ being then the solution to be found. In other words, the relation R between the 2 first images and the 3rd one in a row is the same whatever the row. This assumption is also valid for columns: the relation R between the 2 first images and the 3rd one in a column is the same whatever the column. The natural way to consider a relation between 2 images and a third one is to assume the third one is a combination T of the 2 first ones, where T is an operator $X \times X \to X$, defined componentwise. T has to be defined on 9 pairs. Observing the first and second complete rows, the first and second complete columns of the matrix gives us a complete definition of operator T: For each row or column we get 4 equations:

1st row $T(n, n) = n, T(d, d) = s, T(d, n) = d, T(n, s) = s$;
2nd row $T(d, d) = s, T(s, d) = n, T(d, s) = n, T(n, s) = s$;
1st col. $T(n, d) = d, T(d, s) = n, T(d, d) = s, T(n, n) = n$;
2nd col. $T(n, d) = d, T(d, d) = s, T(n, s) = s, T(s, s) = d$.

This defines T entirely, and we obtain $im9 = ssdd$ since $T(d, d) = s, T(n, s) = s, T(s, s) = d, T(n, d) = d$, leading to the unique solution opposite:

This example shows the creative capabilities of AP-based inference, when employing a feature-based representation (symbolic). Remarkably, this approach has been proved effective on a series of 36 advanced RPM tests (Correa, Prade, and Richard 2012; Correa Beltran, Prade, and Richard

2016); in 16 cases, the approach yields the good result even when applied at a granular pixel level.

Note that APs are here of the form $(x, y) : \mathcal{T}(x, y) :: (x', y') : \mathcal{T}(x', y')$ where $\mathcal{T}$ applies to pairs of vectors and returns a vector. Such APs are close to APs of the form $x : f(x) :: x' : f(x')$, extensively studied by (Hofstadter and Fluid Analogies Research Group 1995), which can be related to Boolean APs (Barbot et al. 2019). In this example, $\mathcal{T}$ is completely defined through the first 8 cells of the matrix. But similar examples where $\mathcal{T}$ is only partially defined would lead to several solutions. This form of creativity is a "novel generation fitted to the constraints of a particular task" (Mayer 1998), as it is usually the case with analogical reasoning (Green et al. 2012).

The RPM tests are another example of the power of analogical proportions-based inference. This approach clearly departs from the ones based on the structure mapping theory (Gentner 1983; Falkenhainer, Forbus, and Gentner 1989), or variants of it (Keane, Ledgeway, and Duff 1994) including the CWSG ("copy with substitution and generation") inference algorithm (Holyoak 2005), which have also been applied to RPM problems, e.g., (Lovett, Forbus, and Usher 2010), or (Kunda, McGreggor, and Goel 2013; Shegheva 2018). But these approaches require to have potential solutions at our disposal, which is not very much in line with the idea of creativity. See (Correa Beltran, Prade, and Richard 2016) for a comparative discussion.

Concluding remarks

This paper has reported a preliminary study on the creative power of APs, showing encouraging results. In her pioneering work (Boden 2004) distinguishes between three forms of creativity: *combinational, exploratory*, and *transformational*. Combinational creativity is the result of combination of familiar ideas, while the other two operate on a conceptual space, the first exploring and the second transforming it. Our work, goes beyond simple combination of vectors, instead we propose a mechanism based on APs, of changing a vector $\vec{c}$ into a vector $\vec{d}$, once the *appropriate context* of another pair $(\vec{a}, \vec{b})$ is present in the environment, subscribing thus to *exploratory* creativity, thanks to the numerous pairs $(\vec{a}, \vec{b})$ and pivots $\vec{c}$ that can be used in general.

The results show that our proposed method is able to identify the existence of animals that one would not think existed, such as animals that lay eggs and breastfeed their children (platypus) or ones that ovoviviparously give birth to their children but do not breastfeed them (scorpions).

Of course, we wanted to examine how this approach compares when it comes to embeddings obtained by various language models. We thus examined both static embeddings, using GloVe and contextual embeddings using BERT. As far as GloVe results are concerned, we have seen that our approach is able to propose "new" animals with up to 83.44% $P@8$ of the proposed animals in the original database, while even results of $P@2$ exceed the Boolean approach. Those results are to be taken only as an indication of the potential of this approach. As (Gladkova, Drozd, and Matsuoka 2016) have shown, GloVe works really well when it comes to simple analogy datasets, such as the one proposed by (Mikolov et al. 2013), but only 30% of the analogies are captured in their new BATS corpus. Recent evidence (Webb, Holyoak, and Lu 2023) show that more advanced models based on Transformer technologies are capable of identifying analogies in a manner that is comparable to humans. We thus opted to examine how contextual embeddings based on Transformer architectures, such as BERT compare to our approach. We thus provided natural language descriptions of animals and calculated sentence embeddings for each one using sBERT. We performed two series of experiments, one containing the name of the animal and one that does not contain the name of the animal. Although this approach can indeed recover idiosyncratic animals (when the name of the animals are provided) such as platypuses and scorpions, they have very little overlap with the symbolic approach. The preliminary nature of our study did not allow us to go into a more fine grained evaluation of the level of creativity of our proposed method (Ritchie 2007) something that we leave for future work.

Generally speaking, generative reasoning by means of APs does not offer any guarantee on the practical value of the results, or their serendipity. Perhaps, one may take inspiration from measures used in recommender systems (Kotkov, Wang, and Veijalainen 2016; Kaminskas and Bridge 2016), for evaluating serendipity in creativity. Yet, AP-based reasoning does not remain in the vicinity of what is known, as illustrated in the examples of this paper, and may yield results outside the box. When dealing with Boolean or nominal representations the results could be explained, and as shown in the Raven example, there is no need of choosing between candidate results.

Finally, we would like to note that in the experiments with the Zoo dataset, we benefited from the knowledge of existing animals, but such information is rarely available for judging creativity. In future work we would like to explore how automatic extraction from text of such features can influence the proposed creative process, in conjunction with a more rigorous evaluation of it.

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