Steps Toward Quantum Computational Creativity

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Abstract

At the heart of creativity is the forging of new concept combinations and the adapting of existing ideas to new situations. However, these processes have resisted mathematical description; concepts violate the rules of classical logic when they interact, e.g., concept combinations can exhibit emergent features not possessed by their constituent concepts. These challenges can be addressed using the quantum cognition framework, wherein nonclassical behavior is described in terms of superposition, entanglement, and interference. While in classical probability theory events are drawn from a common sample space, in quantum models events are defined only with respect to a measurement, or (in quantum cognition) a context, and the probabilities reflect the underlying reality. The measurement (or context) causes collapse from a superposition state to a definite eigenstate. The paper explains how creativity can be modeled using quantum cognition approach with an illustrative example, and discuss how the approach could be implemented computationally. Quantum computing is widely expected to revolutionize many fields in the near future through immense increases in speed and computing power. The time may be ripe to explore the potential of quantum computational creativity.

Introduction

Though creativity is a vast and multifaceted topic (Jordanous and Keller 2016), at its core is the generation of new concept combinations (Estes and Ward 2002). However, modelling concept combination turns out not to be straightforward; there is extensive evidence that people use conjunctions and disjunctions of concepts in ways that violate the rules of classical (including fuzzy) logic; *i.e.*, concepts interact in ways that are non-compositional (Aerts, Aerts, and Gabora 2009; Estes and Ward 2002; Hampton 1988; Osherson and Smith 1981). This noncompositionality is observed in *exemplar typicalities* (*e.g.*, although people do not rate 'guppy' as a typical `PET`, nor a typical `FISH`, they rate it as a highly typical `PET FISH`), as well as *properties* (*e.g.*, although people do not rate 'talks' as a characteristic property of `PET` or `BIRD`, they rate it as characteristic property of `PET BIRD`). This problem has made concepts resistant to mathematical description, and plagued efforts to model how new meanings emerge when people combine concepts and words into larger semantic units such as conjunctions, phrases, or sentences.

One study of this phenomenon analyzed data on the relative frequency of membership of specific exemplars of general categories or concepts, as well as of conjunctions of them (Hampton 1988). For example, participants were asked whether an exemplar such as `Mint` is a member of `FOOD`, whether it is a member of `PLANT`, and whether it is a member of `FOOD AND PLANT`. For several items, participants assessed the examplar as more strongly a member of `FOOD AND PLANT` than of either of the two component concepts `FOOD` and `PLANT` alone. The relative frequency of membership for `Mint`, for example, was 0.87 for the concept `FOOD`, 0.81 for the concept `PLANT`, and 0.9 for the conjunction `FOOD AND PLANT`. It is difficult to conceive of a classical probability model that could encompass this finding, and it was proven that no such model exists, but that it is possible to describe this using a quantum probability model (Aerts 2009). Findings such as this suggest that a quantum approach could prove useful in computational creativity.

This paper outlines the rationale for a quantum approach to modeling creativity, and illustrates the approach using a specific example. It then discusses possible ways to computationally implement the approach. A glossary of terms is provided at the end. Note that the quantum approach does not assume that anything at the quantum level of subatomic particles affects cognition (and in this sense, it is somewhat unfortunate that it has come to be called the quantum approach). It merely uses a generalization of mathematics that was first applied to quantum mechanics.

Rationale for the Quantum Approach

Mental states involving uncertainty, ambiguity, and contextuality figure prominently in creative cognition, and quantum formalisms are uniquely suited to the formal description of such states. This is because a quantum system can be in a *superposition state*, which has the potential to transition into, or (in the quantum jargon) *collapse to* different states depending on how it is measured, or (in quantum cognition), the perspective, or *context*, from which it is considered. For example, consider the situation in which a farmer wonders what to do with an old tire. If he encounters a horse, he might consider the concept `TIRE` in the context <u>see horse</u>, which might lead him to invent a `TIRE BOWL`, i.e., a bowl for his horse (Figure 1). However, if he considers TIRE in the context see child, he might be more likely to make a TIRE SWING. Much as a qubit is not in a specific state (neither 0 nor 1) until a gate causes it to collapse to either 0 or 1, the mental state of wondering what to do with the old tire encompasses multiple possible ideas for reuses of the old tire, and the context influences its 'collapse' to one of them or another. Since the superposition state of a concept incorporates these different possible contexts and outcomes, the quantum approach appears to be better-suited than classical approaches to capture the open-endedness of creativity.

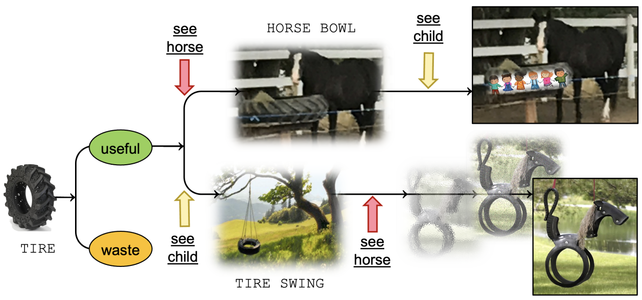

Figure 1: When the context see horse comes before see horse, the old tire is more likely to be used as a horse bowl (top). When the order of contexts is reversed, it is more likely to be used as a horse-shaped tire swing (bottom).

Accordingly, formalisms first used to model situations of ambiguity and contextuality in quantum mechanics (Khrennikov 2010; Wang et al. 2013). have been used to modeled many phenomena relevant to creativity, including semantic spaces and the combination of words and concepts (Aerts 2009; Aerts and Gabora 2005; Gabora and Aerts 2002; Bruza et al. 2009; 2015; Clark, Coecke, and Sadrzadeh 2008; Coecke, Sadrzadeh, and Clark 2010; Lewis, Marsden, and Sadrzadeh 2020), similarity and memory (Pothos and Busemeyer 2022; Nelson et al. 2013), information retrieval (Van Rijsbergen 2004; Melucci 2008), decision making and probability judgement errors, including order effects (Busemeyer, Wang, and Townsend 2006; Busemeyer et al. 2011; Mogiliansky, Zamir, and Zwirn 2009; Yukalov and Sornette 2009) language and text perception (Aerts and Beltran 2020; Surov et al. 2021), cultural evolution (Gabora and Aerts 2009; Gabora, Scott, and Kauffman 2013), tonal attraction in music (Beim Graben and Blutner 2019), and humor (Gabora and Kitto 2017). There have also been findings that cognitive processes exhibit signature features of quantum structure such as superposition, entanglement, and interference (Aerts 2009; Aerts et al. 2012; 2016; Busemeyer and Bruza 2012; Surov et al. 2019).[1]

Brief Outline of the Quantum Approach

Before applying the quantum approach to creativity, we briefly outline how quantum probability differs from classical probability. Classical probability describes events by considering subsets of a common sample space (Isham 1995). That is, considering a set of elementary events, we find that some event e occurred with probability p_e. Classical probability arises due to a lack of knowledge on the part of the modeller. The act of measurement merely reveals an existing state of affairs; it does not interfere with the results. In contrast, quantum models use variables and spaces that are defined (sometimes implicitly) with respect to a particular measurement. Measurements (or contexts) directly influence quantum systems, imposing definite states that may not have been present beforehand (Freedman and Clauser 1972).

In the quantum formalism, the *state* Ψ representing some aspect of interest in our system is written as a linear superposition of a set of possible states referred to as *basis states* $\{\phi_i\}$ in a *Hilbert space*, denoted $\mathcal{H}$, which allows us to define notions such as distance and inner product. In creating this superposition, we weight each basis state with an amplitude term, denoted a_i, which is a complex number representing the contribution of a component basis state ϕ_i to the state Ψ. Hence $\Psi = \sum_i a_i \phi_i$. The probability that the state changes to that basis state upon measurement is $|a|^2$. This non-unitary change of state is called *collapse*, which is modeled as a projection.

The choice of basis states is determined by the value being measured, termed the *observable*, $\hat{O}$. In quantum mechanics, the observables are physical quantities such as position or momentum values (but as we shall see, in quantum cognition they can be, for example, specific instantiations of a concept in a particular context). The potential measurement outcomes o_i correspond to states of the entity of interest. These resultant states of our measurement (or context), are the basis states of the Hilbert space, thus they shape how we model the entity to be measured, and its possible outcomes o_i. The basis states corresponding to an observable outcome are referred to as *eigenstates*. Observables are represented by operators.[2] Upon *measurement*, the state of the entity is projected onto one of the basis states.

It is also possible to describe combinations of two entities within this framework, and to learn about how they might influence one another, or not. Consider two entities A and B with Hilbert spaces $\mathcal{H}_\mathcal{A}$ and $\mathcal{H}_\mathcal{B}$. We may define a basis $|i\rangle_A$ for $\mathcal{H}_\mathcal{A}$ and a basis $|j\rangle_B$ for $\mathcal{H}_\mathcal{B}$, and denote the amplitudes associated with the first as a_i^A and the amplitudes associated with the second as a_j^B. The Hilbert space in which a composite of these entities exists is given by the tensor product $\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$. The most general state in $\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}$ has the form

$$|\Psi\rangle_{AB} = \sum\nolimits_{i,j} a_{ij} |i\rangle_A \otimes |j\rangle_B \tag{1}$$

This state is separable if $a_{ij} = a_i^A a_j^B$. It is inseparable, and therefore an entangled state, if $a_{ij} \neq a_i^A a_j^B$.[3]

[1] The quantum approach may be related to the signal processing approach to meaning generation, and hence creativity, based on spectral modelling of brain activity (Wiggins 2020).

[2] Specifically, Hermetian operators, which are defined on a complex inner product space, but we do not go into that here.

[3] It has been argued that the quantum field theory procedure, which uses Fock space, gives a superior internal structure for modelling concept combination (Aerts 2009). Fock space is the direct sum of tensor products of Hilbert spaces, so it is also a Hilbert space. For simplicity, we omit such refinements here.

Quantum Cognition and its Application to Creativity

We first outline in general terms how the quantum framework is adapted to cognition, and then apply it to creativity using a specific example. The set of possible states of a mental construct, such as a particular concept, is given by Σ. The amplitude term associated with a basis state represented by a complex number coefficient a_i gives a measure of how likely a given change of state is. The basis states represent possible instantiations of the concept. States are represented by unit vectors, and all vectors of a decomposition have unit length, are mutually orthogonal, and generate the whole vector space, thus $\sum_i |a_i|^2 = 1$. Self-adjoint operators[4] are used to define context-specific subspaces. The context causes the state of concept to collapse to one of its eigenstates. The role of the observable is played by the detectable changes to the . Thus, we model change in the concept under a specific context by collapse to a new state.

Each possible form of a concept represented by a particular basis state can be broken down into a set $f_i \in \mathcal{F}$ of features (or properties), which may be weighted according to their relevance with respect to the current context. The *weight* (or renormalized applicability) of a certain property given a specific state of the concept $|p\rangle$ and a specific context $c_i \in \mathcal{C}$ is given by ν. For example, $\nu(p, f_1)$ is the weight of feature f_i for state p. Thus ν is a function from the set $\Sigma \times \mathcal{F}$ to the interval $[0, 1]$. We write:

$$\begin{aligned} \nu : \Sigma \times \mathcal{F} &\rightarrow [0,1] \\ (p, f_i) &\mapsto \nu(p, f_i) \end{aligned} \tag{2}$$

A function μ describes the transition probability from one state to another under the influence of a particular context. For example, $\mu(q, e, p)$ is the probability that state p under the influence of context e changes to state q. Mathematically, μ is a function from the set $\Sigma \times \mathcal{C} \times \Sigma$ to the interval $[0, 1]$, where $\mu(q, e, p)$ is the probability that state p under the influence of context e changes to state q. We write:

$$\begin{aligned} \mu : \Sigma \times \mathcal{C} \times \Sigma &\rightarrow [0,1] \\ (q, e, p) &\mapsto \mu(q, e, p) \end{aligned} \tag{3}$$

Thus our quantum model consists of the 3-tuple $(\Sigma, \mathcal{C}, \mathcal{F})$, and the functions ν and μ.

Let us now make this more concrete using the example of a farmer wondering what to do with an old tire. The state of TIRE, represented by vector $|p\rangle$ of length equal to 1, is a linear superposition of basis states in a complex Hilbert space $\mathcal{H}$ which represent possible states (instances, interpretations, or types) of this concept, including typical states such as SNOW TIRE, and atypical ones such as TIRE SWING. The different states of TIRE can be described as different subspaces into which TIRE can be projected, and thereby, experienced as meaningful. Our knowledge of the possible uses, or affordances, of TIRE comes to us by way of its projections into these subspaces.

[4] Unlike Hermetian operators, self-adjoint operators are defined over the real or complex numbers.

For simplicity, let us suppose that the farmer's initial conception of TIRE is a superposition of only two possibilities (Figure 2). The possibility that the tire is considered *useful* is denoted by the unit vector $|u\rangle$. The possibility that it should be discarded as *waste* is denoted by the unit vector $|w\rangle$. The state of the concept TIRE is denoted $|t\rangle$. Their relationship is given by the equation

$$|t\rangle = a_0|u\rangle + a_1|w\rangle, \tag{4}$$

where a_0 and a_1 are the amplitudes of $|u\rangle$ and $|w\rangle$ respectively in the farmer's mind. States are represented by unit vectors and all vectors of a decomposition such as $|u\rangle$ and $|w\rangle$ have unit length, are mutually orthogonal and generate the whole vector space; thus $|a_0|^2 + |a_1|^2 = 1$.

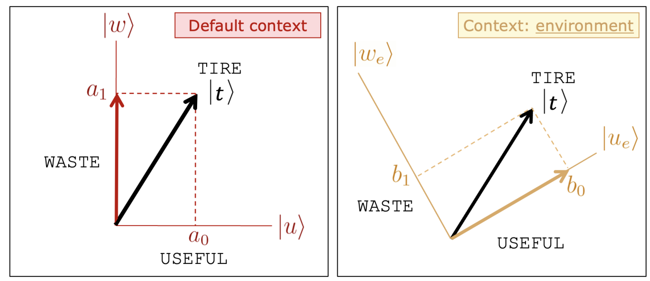

Figure 2: Left: In the default context, TIRE likely collapses to projection vector $|w\rangle$ which represents that it is waste, so $a_0 < a_1$. Right: In the context environment, it likely collapses to orthogonal projection vector $|u\rangle$ which represents that it is useful, so $b_0 > b_1$.

Note that, in someone else's mind a_0 and a_1 might be different (as epitomized in the saying "one person's trash is another person's treasure"). Indeed, if the farmer sees a recycle sign, and thinks of TIRE in the context environment, he himself may feel inspired to find a creative reuse for the tire. Consider the situation in which this is indeed what happens. If the farmer has a horse, the context see horse might be a member of the set $\mathcal{C}$ of possible contexts that could influence the subsequent state of the concept TIRE. The concept TIRE in the context see horse is denoted $|t_h\rangle$.

Activation of the set $\mathcal{L}$ of properties of TIRE, *e.g.,* the property 'weather resistant' denoted f_1, spreads to other concepts for which these properties are relevant. Possible items that must be weather-resistant that could be made from the tire, and thus possible states of $|p_h\rangle$, are (1) a bowl for the horse, or (2) a saddle. We denote HORSE BOWL and TIRE SADDLE $|l\rangle$ and $|e\rangle$, respectively, and the corresponding possible states of TIRE are denoted $|l_h\rangle$ and $|e_h\rangle$. Thus, the restructured conception of TIRE in the context of see horse is given by

$$|t_h\rangle \ = b_0|u_h\rangle + b_1|w_h\rangle \tag{5}$$

where

$$|u_h\rangle = b_2|t_h l_h\rangle + b_3|t_h e_h\rangle + b_4|t_h s_h\rangle, \tag{6}$$

and where $|t_h l_h\rangle$ and $|t_h s_h\rangle$ represent the possibility that he thinks of HORSE BOWL and TIRE SADDLE respectively, and $|t_h\rangle$ represents the possibility that even in the context see horse the farmer thinks of the idea TIRE SWING.

Consider the set of strongly weighted properties of SADDLE, such as 'made of leather' denoted f_2 and 'has stirrups', denoted f_3. Because 'made of leather' and 'has stirrups' are not properties of TIRE, $\nu(t, f_2) << \nu(e, f_2)$, and similarly $\nu(t, f_3) << \nu(e, f_3)$. Therefore, $|b_4|$ is small. However, consider the property of bowl 'has curved edges to keep food in', denoted f_4. Since the curved edges of a tire could stop horse food from falling out, $\nu(t, f_4) \approx \nu(l, f_4)$. Therefore, $|b_3|$ is large. Thus $\mu(l, h, t) >> \mu(e, h, t)$. In the context see horse, the concept TIRE is more likely to collapse to HORSE BOWL. Note that HORSE BOWL has the emergent property, 'holds horse food,' which is a property of neither TIRE nor BOWL. We can model the emergence of new properties by describing TIRE BOWL as an entangled state of the concepts TIRE and BOWL. HORSE BOWL is thereafter a new state of both concepts TIRE and BOWL. Entanglement introduces interference of a quantum nature, and hence the amplitudes are complex numbers (Aerts 2009).

We now consider the contexts see horse and see child, and for simplicity we consider only two possible outcomes for each, HORSE BOWL and TIRE SWING. This could be depicted in an analogous manner to Figure 2, with the contexts default context and environment replaced by see horse and see child, and USEFUL and WASTE replaced by TIRE SWING and HORSE BOWL on the x and y axes respectively. Once again, the context influences the probabilities associated with each reuse idea. Interestingly, as depicted in Figure 1, if *both* contexts are encountered, the final creative outcome depends on the order in which the contexts are encountered. If see horse is encountered first, the thought trajectory likely goes the HORSE BOWL route, but if the child is encountered first, it likely goes the TIRE SWING route, culminating in HORSE TIRE SWING. Such order effects are accommodated in quantum formalism because projection to subspace a_1 then $b_1 \neq$ projection to subspace b_0 then a_0.

The TIRE example shows that a quantum cognition approach to concept interactions, which has been shown to be consistent with human data (Aerts 2009; Aerts et al. 2016), can model the restructuring of concepts during the honing of a creative idea.

Quantum Computational Creativity

Quantum cognition could be incorporated into computational creativity building on existing computational quantum cognition models. The quantum Bayesian network (QBN) combines classical Bayesian networks with quantum probability theory to represent and model human decision-making under uncertainty (Low, Yoder, and Chuang 2014). QBNs have been useful for explaining cognitive biases such as the conjunction fallacy, but more promising routes for modeling creativity are quantum machine learning (Biamonte et al. 2017) or the quantum associative memory approach (Ventura and Martinez 2000), the latter of which proposes that human memory retrieval is influenced by quantum-like interference effects that can account for context-dependent memory. It has been proposed that while such interference effects may have a disruptive effect on retrieval, they enable the fusion of seemingly unrelated context-dependent concepts and ideas that lie at the core of creativity (Gabora and Ranjan 2013). This suggests that such interference effects may be important for computational creativity.

Conclusions

This paper discussed the rationale for bridging quantum cognition and computational creativity, and outlined key steps toward the realization of such a move. A quantum computational creativity model is only as accurate as the number of basis states, properties, and contexts it includes, but with the advent of large language models, this becomes less prohibitive. The approach incorporates the ongoing interaction between potentiality (superposition state) and actualization (eigenstate), and it is this capacity of a quantum system to exist in a superposition of multiple states that lies behind the speed and power of quantum computing, and the widespread belief that it could revolutionize many aspects of our lives. It is widely believed that quantum computing will have a near-term revolutionary impact on many fields, thus, the time seems ripe for exploring its incorporation into computational creativity.

Appendix A: Definitions

Amplitude: A complex number similar to a probability value that gives the likelihood of a particular quantum state. Amplitudes can interfere (constructively or destructively).

Collapse: The change when a quantum system is measured, from a superposition of states to a single definite state.

Eigenstate: A state associated with a particular observable, or context, that has a definite value when measured.

Entanglement: A phenomenon wherein two or more quantum structures are linked—even if widely separated—such that the state of one cannot be described independently of the others, a change to one instantly affects the others.

Hilbert space: A mathematical vector space for describing quantum states and their dynamical evolution.

Interference: The phenomenon wherein waves associated with different quantum possibilities overlap and interact, either constructively, such that they amplify each other, or destructively, such that they cancel each other out.

Observable: A measurable quantity represented by a mathematical operator that acts on a quantum state.

Qubit (short for 'quantum bit'): The fundamental unit of quantum information. Unlike the classical bit, the basic unit of classical computing, which can be either a 0 or a 1, a qubit can exist in a superposition of both 0 and 1 simultaneously.

Superposition: Unlike classical systems, where objects have definite properties, a quantum system can be in a combination of different states at the same time. This combined state is referred to as a superposition.

Acknowledgments

This research was funded by the Natural Sciences and Engineering Research Council of Canada (NSERC) grant number GR026749 AWD-024382. Thanks to former student Haley Ramier for the photograph of a horse-shaped tire swing.

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